

Minijet transverse spectrum in high-energy hadron-nucleus collisions

Alberto Accardi¹ and Daniele Treleani²

*Dipartimento di Fisica Teorica, Università di Trieste,
Strada Costiera 11, I-34014 Trieste*

and

*INFN, Sezione di Trieste
via Valerio 2, I-34127 Trieste*

Abstract

Hadron-nucleus collisions at LHC energies are studied by including explicitly semi-hard parton rescatterings in the collision dynamics. Under rather general conditions, we obtain explicit formulae for the semi-hard cross-section and the inclusive minijet transverse spectrum. As an effect of the rescatterings the spectrum is lowered at small p_t and is enhanced at relatively large transverse momenta, the deformation being more pronounced at increasing rapidity. Its study allows to test the proposed interaction mechanisms and represents an important baseline to examine nucleus-nucleus collisions.

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¹E-mail: accardi@ts.infn.it

²E-mail: daniel@ts.infn.it

1 Introduction

Given the rapid growth of the hard cross-section in hadronic and nuclear collisions [1], the typical inelastic event will be dominated by the perturbative regime at very high energies so that, at the LHC, one may expect to be able to derive global features of the inelastic interaction by perturbative methods. Such a capability, unavoidably limited to a restricted number of physical observables, implies however a few non trivial improvements in the understanding of the mechanisms operating in the interaction process. To be estimated in a sensible way, different physical quantities may in fact need a different degree of understanding of the interaction dynamics, since many details of the process may be of little relevance for some observables, while they may be essential for other quantities. Identifying and evaluating such physical observables represents a non trivial improvement in our capability of using the perturbative QCD to describe physical processes.

An obvious problem will appear when trying to elaborate along these lines. A perturbative calculation does not introduce any scale in the dynamics, so that in this case the kinematic variables are the quantities which give the dimensionality to the related physical observables. The property is associated to the relatively low rate of events falling in the domain of the canonical fixed- x perturbative QCD. On the other hand the dimensional factor which characterizes the global features of the typical inelastic event is, rather, the hadron (or nuclear) scale. When the perturbative regime dominates a physical observable which represents global features of the inelastic interaction, the hadron (or nuclear) scale should therefore appear also in the corresponding perturbative calculation, presumably introduced through the non-perturbative input. The structure functions, namely the up to now non-perturbative input of basically all perturbative calculations, are on the other hand dimensionless quantities. This implies that the structure functions, in their present form, will no longer be an adequate non-perturbative input when trying to accomplish the program outlined above.

A related aspect is the complexity of the interacting states. The canonical, fixed- x , perturbative QCD approach considers only perturbative processes initiated by a pair of partons. The approach is appropriate in the case of very dilute interacting systems, while it becomes obviously inadequate in a regime with very large parton densities. In the case of a partonic interaction in the black disk limit, the initial configuration is in fact isotropic in transverse space, differently from the final state produced by an interaction initiated by two partons (namely, at the leading order in α_S , two jets back-to-back in p_t), where a direction in the transverse plane is singled out. A natural way to recover the black disk symmetry in the final state, is to include in the interaction perturbative processes initiated by more than two partons (namely, semi-hard parton rescatterings), whose relevant property is to produce many large- p_t jets also at the lowest order in α_S . A non-trivial feature is the associated non-perturbative input. To deal with processes initiated by more than two partons one needs in fact to introduce, as a non-perturbative input, the many-body structure functions, which contain independent informations on the hadron (or nuclear) structure with respect to the one-body structure functions needed to deal with processes initiated by two partons. A basic property is that the n -body structure functions are dimensional quantities, in such a way that when n is larger than one the many-parton initiated processes introduce non-perturbative scale factors in the dynamics

of the interaction in a natural way, allowing one to deal with the problem of dimensionality previously mentioned.

By introducing interactions initiated by many partons one may therefore gain the capability of describing, by means of perturbative QCD, at least a few general properties of the typical interaction at the energy of the LHC. To pursue such a program one should then *i)* evaluate in perturbative QCD processes involving many partons in the initial state, *ii)* face the problem of the unknown non perturbative input and develop a strategy in that respect, and *iii)* study the infrared problem by finding observable quantities which are infrared stable. This last step represents the final achievement of the whole program.

The purpose of the present paper is to discuss the case of hadron-nucleus interactions (hA, for brevity). Being intermediate between hadron-hadron (hh) and nucleus-nucleus (AA), hadron-nucleus interactions allow several simplifications in the formalism developed to discuss heavy-ion collisions. In fact, as it will be shown hereafter and differently with respect to the latter case, in the hadron-nucleus instance we will be able to obtain closed analytic expressions for the semi-hard cross-section under rather general conditions. We will then study the inclusive minijet transverse spectrum, which is related in a direct way to the underlying dynamics and is therefore an important baseline for the study of nucleus-nucleus collisions.

Beside its intrinsic interest, inclusion of semi-hard rescatterings in the computation of the transverse spectrum has been advocated by many authors [2, 3, 4, 5, 6] as the basic mechanism underlying the Cronin effect [7], namely the deformation of the hadron p_t -spectra in nuclear collisions as compared with the expectations of a single large- p_t production mechanism. Multiple parton collisions have also been related to higher-twist parton distributions [8, 9, 10]. A non-perturbative study of the transverse spectrum in hA collisions in the framework of the McLerran-Venugopalan model for nuclear and hadronic collisions was presented in [11].

Another reason of interest in hadron-nucleus collisions is that theoretical models can be tested against experimental data in a situation where further nuclear effects are absent, like, e.g., the formation of a hot and dense medium which can further modify the transverse spectrum via energy-loss [12, 13]. Therefore a detailed understanding of hA collisions represents an important baseline for the generalization to AA collisions [14, 15] and for the discovery of novel physical effects [16].

An explicit approach to semi-hard interactions in heavy ion collisions at the LHC on the lines previously described, has been accomplished, at least partially, with the help of a few simplifying hypotheses. The program has been implemented in [17, 18, 19, 20], and various physical quantities have been evaluated in [21, 22]. The approach relies, to a large extent, on the idea of self-shadowing, which we recall for completeness in Sec. 2. In Sec. 3 we discuss our expression of the semi-hard interaction probability between two colliding partonic configurations. We discuss also the multi-parton distributions, that are studied with a functional formalism and finally combined with the interaction probability to derive the hadron-nucleus semi-hard cross-section. Sec. 4 is devoted to the discussion of the inclusive minijet transverse spectrum, with particular emphasis on the mechanism of subtraction of infrared divergences, which is explicitly implemented in our approach. Results of numerical evaluations of the inclusive spectra of minijets in hadron-nucleus

collisions are presented in Sec. 5. The last section is devoted to the concluding summary.

2 Self-shadowing

To face the problem of unitarity corrections we make use of the self-shadowing property of the hard component of the interaction. For the sake of completeness, in the present paragraph we recall the main points about the self-shadowing cross-sections in hadron-nucleus interactions [23].

Let's consider the inelastic hadron-nucleus cross-section $(\sigma_{in})_A$, whose expression may be expanded, in the Glauber approach, as a binomial probability distribution of inelastic nucleon-nucleon collisions:

$$\begin{aligned} (\sigma_{in})_A &= \int d^2\beta \left[1 - \left(1 - \sigma_{in}\tau(\beta) \right)^A \right] \\ &= \int d^2\beta \sum_{n=1}^A \binom{A}{n} (\sigma_{in}\tau(\beta))^n (1 - \sigma_{in}\tau(\beta))^{A-n} \end{aligned} \quad (2.1)$$

In Eq. (2.1) $\tau(\beta)$ is the nuclear thickness function, which depends on the impact parameter β and is normalized to one, A is the atomic mass number and σ_{in} is the inelastic hadron-nucleon cross-section. One may classify all events according to a given selection criterion, which we call \mathcal{C} , while we call \mathcal{N} the events that are not of kind \mathcal{C} . We assume that in a hadron-nucleon collision all events of kind \mathcal{C} contribute to $\sigma_{\mathcal{C}}$, all other events contribute to $\sigma_{\mathcal{N}}$, so that the inelastic hadron-nucleon cross-section may be written as

$$\sigma_{in} = \sigma_{\mathcal{C}} + \sigma_{\mathcal{N}} .$$

One may then ask for the expression of the cross-section $(\sigma_{\mathcal{C}})_A$ to produce events of kind \mathcal{C} in a collision of a hadron against a nuclear target. Then, to obtain $(\sigma_{\mathcal{C}})_A$, one may express $(\sigma_{in})^n$ in Eq. (2.1) as a binomial sum of “elementary” events of kind \mathcal{C} and of kind \mathcal{N} :

$$\sigma_{in}^n = (\sigma_{\mathcal{C}} + \sigma_{\mathcal{N}})^n = \sum_{k=0}^n \binom{n}{k} \sigma_{\mathcal{C}}^k \sigma_{\mathcal{N}}^{n-k} . \quad (2.2)$$

An interesting case to consider is when the events of kind \mathcal{C} are such that any superposition of elementary events of kind \mathcal{C} , both with events of kind \mathcal{C} and of kind \mathcal{N} , always gives an event of kind \mathcal{C} . In this case, all the terms of the sum in Eq. (2.2), with the only exception of the term with $k = 0$, contribute to $(\sigma_{\mathcal{C}})_A$, which is therefore given by:

$$(\sigma_{\mathcal{C}})_A = \int d^2\beta \sum_{n=1}^A \binom{A}{n} \left[\sum_{k=1}^n \binom{n}{k} \sigma_{\mathcal{C}}^k \sigma_{\mathcal{N}}^{n-k} \right] (\tau(\beta))^n (1 - \sigma_{in}\tau(\beta))^{A-n} .$$

By using the relation

$$\sum_{k=1}^n \binom{n}{k} \sigma_{\mathcal{C}}^k \sigma_{\mathcal{N}}^{n-k} = \sigma_{in}^n - \sigma_{\mathcal{N}}^n ,$$

one obtains:

$$\begin{aligned}
(\sigma_{\mathcal{C}})_A &= \int d^2\beta \sum_{n=1}^A \binom{A}{n} \left[(\sigma_{in}\tau(\beta))^n - (\sigma_{\mathcal{N}}\tau(\beta))^n \right] [1 - \sigma_{in}\tau(\beta)]^{A-n} \\
&= \int d^2\beta \left[(\sigma_{in}\tau(\beta) + 1 - \sigma_{in}\tau(\beta))^A - (\sigma_{\mathcal{N}}\tau(\beta) + 1 - \sigma_{in}\tau(\beta))^A \right] \\
&= \int d^2\beta \left[1 - (1 - \sigma_{\mathcal{C}}\tau(\beta))^A \right] \\
&= \int d^2\beta \sum_{n=1}^A \binom{A}{n} [\sigma_{\mathcal{C}}\tau(\beta)]^n [1 - \sigma_{\mathcal{C}}\tau(\beta)]^{A-n} .
\end{aligned} \tag{2.3}$$

Notice that, in spite of the fact that we included superpositions of elementary events of kind \mathcal{C} with events both of kind \mathcal{C} and of kind \mathcal{N} , the nuclear cross-section $(\sigma_{\mathcal{C}})_A$ is obtained by summing all possible multiple hadron-nucleon interactions of kind \mathcal{C} alone with a binomial probability distribution, precisely as $(\sigma_{in})_A$ is obtained by a binomial distribution of hadron-nucleon inelastic interactions. This relation states the self shadowing property of the events of kind \mathcal{C} : all unitarity corrections, namely the term $[1 - \sigma_{\mathcal{C}}\tau(\beta)]^A$ in the third line of Eq. (2.3), are expressed by means of the cross-section $\sigma_{\mathcal{C}}$ only. However, this does not mean that $(\sigma_{\mathcal{C}})_A$ doesn't contain events of kind \mathcal{N} , but rather that they are irrelevant to obtain $(\sigma_{\mathcal{C}})_A$. The property that an event of kind \mathcal{C} remains of kind \mathcal{C} even after any number of events of kind \mathcal{N} translates into the disappearance of $\sigma_{\mathcal{N}}$ in the nuclear cross-section $(\sigma_{\mathcal{C}})_A$.

Given the discussion above, the only part of the nuclear interaction that still misses is the cross-section for elementary events of kind \mathcal{N} alone. It can be obtained by considering the following difference

$$\begin{aligned}
\frac{d(\sigma_{in})_A}{d^2\beta} - \frac{d(\sigma_{\mathcal{C}})_A}{d^2\beta} &= [1 - \sigma_{\mathcal{C}}\tau(\beta)]^A - [1 - (\sigma_{\mathcal{C}} + \sigma_{\mathcal{N}})\tau(\beta)]^A \\
&= [1 - \sigma_{\mathcal{C}}\tau(\beta)]^A \times \left\{ 1 - \left[1 - \frac{\sigma_{\mathcal{N}}\tau(\beta)}{1 - \sigma_{\mathcal{C}}\tau(\beta)} \right]^A \right\} \\
&= [1 - \sigma_{\mathcal{C}}\tau(\beta)]^A \times \sum_{k=1}^A \binom{A}{k} \left(\frac{\sigma_{\mathcal{N}}\tau(\beta)}{1 - \sigma_{\mathcal{C}}\tau(\beta)} \right)^k \left(1 - \frac{\sigma_{\mathcal{N}}\tau(\beta)}{1 - \sigma_{\mathcal{C}}\tau(\beta)} \right)^{A-k} ,
\end{aligned} \tag{2.4}$$

which is therefore bounded by $[1 - \sigma_{\mathcal{C}}\tau(\beta)]^A$ (second line of 2.4), namely by the probability of not having any interaction of kind \mathcal{C} at a given impact parameter β . The ratio $\sigma_{\mathcal{N}}\tau(\beta)/[1 - \sigma_{\mathcal{C}}\tau(\beta)]$ is in fact a quantity smaller than one, since $\sigma_{in}\tau(\beta)$, which is equal to $(\sigma_{\mathcal{C}} + \sigma_{\mathcal{N}})\tau(\beta)$, is a probability. It may be understood as the probability of an hadron-nucleon interaction at a given impact parameter, under the condition that no event of kind \mathcal{C} takes place. Hence the last line of Eq. (2.4) shows that after removing all events of kind \mathcal{C} the interaction is expressed by a binomial distribution of events of kind \mathcal{N} .

Finally, we observe that if we compute the average number of hadron-nucleon collisions

of kind \mathcal{C} , $\langle n \rangle (\sigma_{\mathcal{C}})_A$, rather than the cross-section $(\sigma_{\mathcal{C}})_A$, the result is:

$$\begin{aligned} \langle n \rangle (\sigma_{\mathcal{C}})_A &= \int d^2\beta \sum_{n=1}^A n \binom{A}{n} (\sigma_{\mathcal{C}}\tau(\beta))^n (1 - \sigma_{\mathcal{C}}\tau(\beta))^{A-n} \\ &= \int d^2\beta \frac{d}{d\gamma} \sum_{n=1}^A \binom{A}{n} (\sigma_{\mathcal{C}}\tau(\beta)\gamma)^n (1 - \sigma_{\mathcal{C}}\tau(\beta))^{A-n} \Big|_{\gamma=1} \\ &= A\sigma_{\mathcal{C}} \end{aligned}$$

Notice that the average number of interactions of kind \mathcal{C} is expressed by the single-scattering term, without any unitarity correction.

3 Semi-hard cross-section

In this section we want to represent the semi-hard hadron-nucleus cross-section analogously to the self-shadowing cross-section (2.3), but considering as elementary objects the partons instead of the nucleons. Indeed, the hard component of the interaction satisfies the requirements of the self-shadowing cross-sections if one assumes that a parton which has undergone interactions with large momentum exchange can always be recognized in the final state. An immediate difference with respect to the previous case is that now there is no upper bound on the number of partons that can take part in the collision. Because of self-shadowing all unitarity corrections to the semi-hard cross-section will be therefore expressed by means of the semi-hard partonic cross-section only, so that one doesn't need to make any commitment on the soft component when only the semi-hard part of the interaction is of interest. Self-shadowing allows moreover to control also the soft component of the interaction by perturbative means, since that contribution is limited to a fraction of the cross-section proportional to the probability of not having any hard interaction at all (see Eq. 2.4). Obviously the unavoidable restriction of all considerations done by perturbative means is that those are limited to partonic final states, whose properties will hopefully survive hadronization.

To represent the interaction between hadrons and nuclei in terms of partonic interactions, each one with relatively large momentum exchange, one needs to write the cross-section for a given non-perturbative input, namely for a definite partonic configuration of the two interacting objects. Then, as a perturbative input, one needs to write the probability of having at least one semi-hard interactions between the two configurations of partons. We discuss these two inputs in the next two subsections and in Sec. 3.3 we combine them to obtain the hadron-nucleus cross-section.

3.1 Perturbative input: semi-hard rescatterings

In a processes involving many partons in the initial state, an important distinction is between connected and disconnected hard interactions. A hard process may in fact be represented also by a disconnected hard amplitude, since the overall interaction process may still be connected by the soft component of the interaction. The simplest case of a

disconnected process is obviously the one where the hard part is represented by two partonic interactions at the lowest order in perturbative QCD, corresponding to two $2 \rightarrow 2$ parton scatterings. Since all hard collisions are characterized by short transverse distances, disconnected hard processes give rise to a picture of the interaction where the different partonic collisions are all localized in different points in the transverse space, with a transverse distance of the order of the scale of soft interactions [24]. The disconnected component of the hard-interaction leads therefore to a geometrical picture of the process, giving as well some indications on the degrees of freedom characterizing the non perturbative input. The many-body parton distributions need in fact to depend explicitly on the parton transverse coordinates, to identify the partons involved in each given sub-processes with a definite localization in transverse space. Notice that the dependence on the transverse coordinates and the number of partons taking part the interaction are the basic information needed to assign the dimensionality to the many-body structure functions, and therefore to introduce the non-perturbative scale factors in the interaction dynamics.

While the main feature of the disconnected component of the hard amplitude is to give rise to a geometrical picture of the semi-hard interaction, the connected component of the amplitude becomes more and more structured when one approaches the black disk limit, where a single projectile parton may interact with several target partons with large momentum exchange in different directions in transverse space. The simplest possibility of such an interaction was discussed in Ref.[20], where the forward amplitude of the process and all the cuts were derived in the case of a point-like projectile against two point-like targets, in the limit of an infinite number of colors and for $t/s \rightarrow 0$. In this case one finds that the different cuts of the $3 \rightarrow 3$ forward amplitude are all proportional one to another and the proportionality factors are the AGK weights [25]. A consequence is that one may express the three-body interaction as a product of two-body interaction probabilities. The results obtained in that simple case may indicate a convenient approximation of the many-parton interaction probability. One can in fact argue that the many-parton interaction process may be approximated by a product of two-parton interactions, so that one can call the process *re-interaction* or *rescattering*. The whole interaction is therefore expressed in terms of two-body interaction probabilities, precisely as the interaction between two nuclei is expressed in terms of nucleon-nucleon collisions. Hence, given a configuration with n partons of the projectile and l partons of the target, we introduce the probability, $\mathcal{P}_{n,l}$, of having at least one partonic collision, in a way analogous to the expression of the inelastic nucleus-nucleus cross-section [26]:

$$\mathcal{P}_{n,l} = \left[1 - \prod_{i=1}^n \prod_{j=1}^l (1 - \hat{\sigma}_{ij}) \right], \quad (3.1)$$

where $\hat{\sigma}_{ij}$ is the probability of interaction of a given pair of partons i and j . Since the distance over which the hard interactions are localized is much smaller than the soft interaction scale, one may approximate $\hat{\sigma}(x_i x_j; b_i - b_j) \approx \sigma(x_i x_j) \delta^{(2)}(b_i - b_j)$, where x_i and x_j are the momentum fractions of the colliding partons, b_i and b_j their transverse coordinates and $\sigma(x_i x_j)$ is the partonic cross section, whose infrared divergence is cured by introducing a regulator p_0 . For example, p_0 may be the lower cutoff on the momentum exchange in each partonic collision, or a small mass introduced in the transverse prop-

agator to prevent the divergence of the cross-section at zero momentum exchange. The expression of $\mathcal{P}_{n,l}$ is the analogue of Eq. (2.3) and represents the explicit implementation of self-shadowing for the interaction of two partonic configurations.

3.2 Non-perturbative input: multi-parton distributions

In this section we discuss the non perturbative input to the process. To approach the problem in the most general form we use the functional formalism introduced in [19]. At a given resolution, provided by the regulator p_0 , one may find the nuclear (or hadronic) system in various partonic configurations. We call $P^{(n)}(u_1 \dots u_n)$ the probability of a configuration with n -partons (the *exclusive n -parton distribution*) where $u_i \equiv (b_i, x_i)$ represents the transverse coordinate of the i -th parton, b_i , and its longitudinal fractional momentum, x_i . The distributions are symmetric in the variables u_i , and can be obtained from a generating functional defined with the help of auxiliary functions $J(u)$ as follows:

$$\mathcal{Z}[J] = \sum_n \frac{1}{n!} \int J(u_1) \dots J(u_n) P^{(n)}(u_1, \dots, u_n) du_1 \dots du_n ,$$

all infrared divergences are regularized by p_0 , which is implicit in all equations. Probability conservation yields the normalization condition $\mathcal{Z}[1] = 1$. Then, the exclusive n -parton distributions can be obtained by differentiating the generating functional \mathcal{Z} with respect to the auxiliary functions:

$$P^{(n)}(u_1, \dots, u_n) = \frac{\delta}{\delta J(u_1)} \dots \frac{\delta}{\delta J(u_n)} \mathcal{Z}[J] \Big|_{J=0} .$$

A useful representation of \mathcal{Z} may be found by introducing its logarithm, \mathcal{F} , with normalization $\mathcal{F}[1] = 0$, so that

$$\mathcal{Z}[J] = e^{\mathcal{F}[J]} ,$$

and by studying the *inclusive n -parton distribution*, $D^{(n)}$. They can be obtained as functional derivatives of \mathcal{Z} or of \mathcal{F} . Indeed

$$\begin{aligned} D^{(1)}(u) &\equiv P_1(u) + \int P^{(2)}(u, u') du' + \frac{1}{2} \int P^{(3)}(u, u', u'') du' du'' + \dots \\ &= \frac{\delta \mathcal{Z}}{\delta J(u)} \Big|_{J=1} = \frac{\delta \mathcal{F}}{\delta J(u)} \Big|_{J=1} , \\ D^{(2)}(u_1, u_2) &\equiv P^{(2)}(u_1, u_2) + \int P^{(3)}(u_1, u_2, u') du' + \frac{1}{2} \int P^{(4)}(u_1, u_2, u', u'') du' du'' \dots \\ &= \frac{\delta^2 \mathcal{Z}}{\delta J(u_1) \delta J(u_2)} \Big|_{J=1} = \frac{\delta^2 \mathcal{F}}{\delta J(u_1) \delta J(u_2)} \Big|_{J=1} + \frac{\delta \mathcal{F}}{\delta J(u_1)} \frac{\delta \mathcal{F}}{\delta J(u_2)} \Big|_{J=1} , \end{aligned}$$

and so on for higher multi-parton distributions. These relations show that the correlated part, $C^{(n)}$, of the inclusive n -parton distribution (also called *n -parton correlation*) is simply given by differentiation of the generating functional \mathcal{F} :

$$C^{(n)}(u_1, \dots, u_n) = \frac{\delta}{\delta J(u_1)} \dots \frac{\delta}{\delta J(u_n)} \mathcal{F}[J] \Big|_{J=1} ,$$

so that the expansion of \mathcal{F} near $J = 1$ reads:

$$\mathcal{F}[J] = \int \Gamma(u)[J(u) - 1]du + \sum_{n=2}^{\infty} \frac{1}{n!} \int C^{(n)}(u_1 \dots u_n)[J(u_1) - 1] \dots [J(u_n) - 1] du_1 \dots du_n ,$$

where $\Gamma(u) \equiv D^{(1)}(u)$ for consistency with the notation used in previous papers. In this way we have obtained a convenient representation of the generating functional $\mathcal{Z} = \exp[\mathcal{F}]$ in terms of the single parton inclusive distribution, Γ , and of the multi-parton correlations, $C^{(n)}$. In the simplest case where we neglect all the correlations between the partons, namely $C^{(n \geq 2)} = 0$, the generating functional is given by

$$\mathcal{Z}[J] = e^{\int \Gamma(u)[J(u)-1]du} . \quad (3.2)$$

3.3 Hadron-nucleus cross-section

The general expression of the semi-hard cross-section at fixed impact parameter is obtained by folding the interaction probability, Eq. (3.1), with the multi-parton exclusive distributions of the two colliding systems (in our case a hadron, h , and a nucleus of atomic number A):

$$\begin{aligned} \frac{d\sigma_H}{d^2\beta} &= \int \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\delta}{\delta J(u_1 - \beta)} \dots \frac{\delta}{\delta J(u_n - \beta)} \mathcal{Z}_h[J] \Big|_{J=0} \\ &\times \sum_{m=1}^{\infty} \frac{1}{m!} \frac{\delta}{\delta J'(u'_1)} \dots \frac{\delta}{\delta J'(u'_m)} \mathcal{Z}_A[J'] \Big|_{J'=0} \\ &\times \left\{ 1 - \prod_{i=1}^n \prod_{j=1}^m [1 - \hat{\sigma}_{ij}(u, u')] \right\} \prod_{i=1}^n du_i \prod_{j=1}^m du'_j , \end{aligned} \quad (3.3)$$

where β is the impact parameter between h and A . To simplify the notation we introduce the following operators:

$$\delta_i = \int du_i \frac{\delta}{\delta J(u_i - \beta)} \quad ; \quad \delta'_j = \int du_i \frac{\delta}{\delta J'(u'_j)} .$$

Given two functions $f = f(u)$ and $g = g(u)$, the following identity holds:

$$e^{\delta \cdot f} \mathcal{Z}[J + g] = \mathcal{Z}[J + g + f] , \quad (3.4)$$

where $\delta \cdot f = \int du \delta / \delta J(u) f(u)$. In other words, the exponential of the operator δ acts on the generating functional \mathcal{Z} by shifting its argument of the amount f .

In the case of hadron-nucleus interactions one may be allowed to neglect the rescatterings of the partons of the nucleus. Indeed, even at very high center of mass energies the average number of scattering per incoming parton is smaller than the average number

of nucleons along the parton trajectory, except in the very forward rapidity region [21]. With this assumption the interaction probability may be simplified as follows:

$$\left\{1 - \prod_{i,j}^{n,m} [1 - \hat{\sigma}_{ij}] \right\} \simeq \sum_{i,j} \hat{\sigma}_{ij} - \frac{1}{2!} \sum_{i,k} \sum_{j \neq l} \hat{\sigma}_{ij} \hat{\sigma}_{kl} + \frac{1}{3!} \sum_{i,k,r} \sum_{j \neq l \neq s} \hat{\sigma}_{ij} \hat{\sigma}_{kl} \hat{\sigma}_{rs} + \dots \quad (3.5)$$

After contracting Eq. (3.5) with the differentiation operators in Eq. (3.3) one obtains

$$\begin{aligned} \sum_n \frac{1}{n!} \delta_1 \dots \delta_n \sum_{q \geq 1} \frac{(-1)^{q-1}}{q!} \left[\sum_{i=1}^n \delta' \cdot \hat{\sigma}_i \right]^q e^{\delta'} &= \sum_n \frac{1}{n!} \delta_1 \dots \delta_n \left[1 - \exp \left(- \sum_{i=1}^n \delta' \cdot \hat{\sigma}_i \right) \right] e^{\delta'} \\ &= \left\{ 1 - \exp \left[\delta \cdot (e^{-\delta' \cdot \hat{\sigma}} - 1) \right] \right\} e^{\delta + \delta'} . \end{aligned}$$

By using the identity (3.4), the semi-hard cross-section becomes:

$$\frac{d\sigma_H}{d^2\beta} = \left\{ 1 - \exp \left[\delta \cdot (e^{-\delta' \cdot \hat{\sigma}} - 1) \right] \right\} \mathcal{Z}_h[J+1] \mathcal{Z}_A[J'+1] \Big|_{J=J'=0} \quad (3.6)$$

This result is very general and includes all possible parton correlations of both the projectile and the target; the only assumption made is that target partons do not suffer any semi-hard rescattering (we will comment more on this assumption in Sec. 5.1). A meaningful approximation (see Ref.[19]) is to consider the nuclear partons uncorrelated, namely $C_A^{(n \geq 2)} = 0$. Then, by using Eq. (3.2) the cross-section reduces to:

$$\begin{aligned} \frac{d\sigma_H}{d^2\beta} &= \sum_{n=1}^{\infty} \frac{\delta^n}{n!} \sum_{m=0}^n (-1)^{n-m} \binom{m}{n} \mathcal{Z}_A[1 - m\hat{\sigma}] \mathcal{Z}_h[J+1] \Big|_{J=0} = \\ &= 1 - \mathcal{Z}_h \left[e^{-\int \hat{\sigma}(\cdot, u') \Gamma_A(u') du'} \right] . \end{aligned} \quad (3.7)$$

If we neglect also the correlations between the partons of the projectile, we get a further simplification:

$$\frac{d\sigma_H}{d^2\beta} = 1 - \exp \left\{ - \int du \Gamma_h(u - \beta) \left[1 - e^{-\int \hat{\sigma}(u, u') \Gamma_A(u') du'} \right] \right\} . \quad (3.8)$$

Both in Eq. (3.7) and in Eq. (3.8) the cross-section is a function of

$$\begin{aligned} W_h(u, \beta) &= \Gamma_h(u - \beta) \left[1 - e^{-\int \hat{\sigma}(u, u') \Gamma_A(u') du'} \right] \\ &= \Gamma_h(u - \beta) \mathcal{P}_A(u) , \end{aligned} \quad (3.9)$$

which represents the number of projectile partons that have interacted with the target, i.e., the projectile *wounded partons* [17, 19]; we call them *minijets*, even if they did not yet hadronize. $\mathcal{P}_A(u)$ represents the probability that a projectile parton with given $u = (x, b)$ has at least one semi-hard interaction with the target, hence the cross-section is obtained by summing all events with at least one interaction.

One might obtain the average number of wounded partons, Eq. (3.9) by working out directly from Eq. (3.3) the average number of projectile partons which have undergone

hard interactions [17, 19] (a detailed numerical study of this quantity in Pb-Pb collisions at LHC and RHIC energies is presented in [21, 22]). The result, Eq. (3.9), is obtained under the only assumption that all the target partons are uncorrelated. Therefore, $\int du W(u, \beta) = \langle n \rangle d\sigma/d^2\beta$ represents the integrated inclusive cross-section to detect all scattered projectile partons, and takes into account the correlations of the projectile partons at all orders. Of course, the projectile parton correlations appear explicitly in the total hadron-nucleus cross-section. In the simplest case of two-parton correlations one would obtain:

$$\frac{d\sigma_H}{d^2\beta} = 1 - \exp \left\{ - \int du W_h(u - \beta) + \frac{1}{2!} \int du du' \mathcal{P}_A(u) C_h^{(2)}(u - \beta, u' - \beta) \mathcal{P}_A(u') \right\}. \quad (3.10)$$

The effect of correlations on $d\sigma_H/d^2\beta$ is however small, both when unitarity corrections are small (i.e., when the semi-hard parton-parton cross-section is small, so that \mathcal{P}_A and W_h are both of order σ_H) and when they are large (i.e., when σ_H is large, $\mathcal{P}_A \sim 1$ and W_h is large). If, on the other hand, one is looking for correlations, the simplest quantity which depends linearly on $C_h^{(2)}$ is the double-jet inclusive cross-section.

4 Inclusive minijet transverse spectrum

The whole semi-hard hadron-nucleus cross-section results from the superposition of the multiple interactions of the partons of the projectile hadron (which is a dilute partonic system) with the nuclear target (which is a dense partonic system). The inclusive transverse spectrum of the projectile minijets is given by the distribution in p_t of the average number of wounded partons, and is affected by the presence of semi-hard rescatterings [18]. The deformation of the high- p_t hadron spectra, which leads to the Cronin effect, was studied in terms of semi-hard parton rescatterings in [2, 3, 4, 5], where partons that suffered up to two scatterings were included, leading to a good description of the data for pA collisions up to $\sqrt{s} = 39$ GeV/A. However, the two-scattering approximation breaks down at higher energies, except at very high p_t , and the whole wounded parton transverse spectrum is needed. More phenomenological approaches [6, 14], which take into account also intrinsic transverse momentum, model the effects of multiple scattering as an additional Gaussian p_t -broadening for each rescattering suffered by a parton. A random-walk model of the multiple scatterings was proposed in [15].

After the introduction of semi-hard parton rescatterings, integrated quantities like the semi-hard cross-section and the minijet multiplicity show a weak dependence on the infrared cutoff needed to regularize the infrared divergences arising in the perturbative computations [17, 21]. On the contrary, it will be shown that differential quantities like the minijet p_t -spectrum are more sensitive on the detailed dynamics of the interaction and show a stronger dependence on the cutoff, if only logarithmic. To reduce this dependence on the cutoff one needs to improve further the picture of the dynamics by including also gluon radiation in the interaction process. Some steps along this line in the case of deep inelastic electron-nucleus scattering have been presented in [10]. In this paper, however, we neglect the problem of the gluon radiation and we concentrate on the effects of elastic rescatterings.

4.1 Transverse spectrum

We can expand the average number of projectile wounded partons, Eq. (3.9), at a given x and b in a collision with impact parameter β , in the following way:

$$W_h(x, b, \beta) = \Gamma_h(x, b - \beta) \sum_{\nu=1}^{\infty} \frac{\langle n_A(x, b) \rangle^\nu}{\nu!} e^{-\langle n_A(x, b) \rangle}, \quad (4.1)$$

where $\langle n_A(x, b) \rangle \equiv \int dx' \Gamma_A(x', b) \sigma(xx')$ is the average number of scatterings of a projectile parton at a given x and b [19]. The average number of wounded partons is then given by the average number of incoming partons, Γ_h , multiplied by the probability of having at least one semi-hard scattering, which is given by a Poisson distribution in the number of scatterings, ν , with average number $\langle n_A(x, b) \rangle$. Therefore, we can obtain the inclusive differential distribution in p_t by introducing a constraint in the transverse momentum integrals that give the integrated parton-parton cross sections in the expression above:

$$\begin{aligned} \frac{dW_h}{d^2p_t}(x, b, \beta) &= \Gamma_h(x, b - \beta) \sum_{\nu=1}^{\infty} \frac{1}{\nu!} \int \Gamma_A(x'_1, b) \dots \Gamma_A(x'_\nu, b) e^{-\int dx' \Gamma_A(x', b) \sigma(xx')} \\ &\times \frac{d\sigma}{d^2k_1} \dots \frac{d\sigma}{d^2k_\nu} \delta^{(2)}(\mathbf{k}_1 + \dots + \mathbf{k}_\nu - \mathbf{p}_t) d^2k_1 \dots d^2k_\nu dx'_1 \dots dx'_\nu. \end{aligned} \quad (4.2)$$

The limits of integration on x'_i and x' are respectively $xx'_i \geq 4k_i^2$ and $xx' \geq 4p_0^2$, and all the distribution functions are evaluated for simplicity at a fixed scale.

By using the above formula one can study the p_t -broadening of a wounded parton, namely, the square root of the average transverse momentum squared acquired through its path across the nucleus. Consider a single projectile parton with fixed x and b . The probability that it acquires a certain p_t after the collision is given by Eq. (4.2) divided by the number, $\Gamma_h(x, b - \beta)$, of incoming partons:

$$\frac{d\mathcal{P}_A(x, b)}{d^2p_t} = \frac{dW_h(x, b, \beta)}{d^2p_t} \frac{1}{\Gamma_h(x, b - \beta)}.$$

Then, the average transverse momentum squared of a wounded parton is given by

$$\langle p_t^2(x, b) \rangle_A = \frac{\langle \langle p_t^2 \rangle \rangle}{\langle \langle 1 \rangle \rangle},$$

where $\langle \langle f(p_t) \rangle \rangle = \int d^2p_t f(p_t) d\mathcal{P}_A/d^2p_t$. The delta function in Eq. (4.2) tells that at a fixed number, ν , of scatterings we have $\langle \langle p_t^2 \rangle \rangle_\nu = \langle \langle (\sum_{i=1}^\nu \mathbf{k}_i)^2 \rangle \rangle = \langle \langle \sum_{i=1}^\nu k_i^2 \rangle \rangle$. The last equality is due to the azimuthal symmetry of the differential parton-parton cross-sections $\frac{d\sigma}{d^2k_i}$. The resulting expression is symmetric under exchanges of the transverse momenta k_i , so that $\langle \langle p_t^2 \rangle \rangle_\nu = \nu \langle \langle k_i^2 \rangle \rangle$. Then it is immediate to see that

$$\langle p_t^2(x, b) \rangle_A = \frac{1}{\mathcal{P}_A} \int d^2p_t dx' p_t^2 \frac{d\sigma}{d^2p_t}(xx') \Gamma_A(x', b) = \langle p_t^2(x, b) \rangle_1 \frac{\langle n_A(x, b) \rangle}{\mathcal{P}_A(x, b)}, \quad (4.3)$$

where

$$\langle p_t^2(x, b) \rangle_1 = \frac{\int d^2p_t dx' p_t^2 \frac{d\sigma(xx')}{d^2p_t} \Gamma_A(x', b)}{\int dx' \sigma(xx') \Gamma_A(x', b)}$$

is the average transverse momentum squared in a single parton-parton collision. The p_t -broadening of the wounded partons in a hA collision is then given by the p_t -broadening in a single collision multiplied by the average number of rescatterings suffered by a wounded parton. A similar result for the p_t -broadening of a fast parton traversing a nuclear medium was derived in [27]. The p_t -broadening was also studied in different contexts in [9, 10]. Two interesting limits can be considered:

$$\langle p_t^2(x, b) \rangle_A \sim \begin{cases} \langle p_t^2(x, b) \rangle_1 & \text{as } p_0 \rightarrow \infty \\ \langle p_t^2(x, b) \rangle_1 \langle n_A(x, b) \rangle & \text{as } p_0 \rightarrow 0 . \end{cases} \quad (4.4)$$

Since the minijet yield is dominated by transverse momenta of the order of the cutoff, these two limits say roughly that the minijets at high p_t (i.e., high p_0 in Eq. (4.4)) suffer mainly one scattering. On the contrary, at low p_t (i.e., low p_0 in Eq. (4.4)) they undergo a random walk in the transverse momentum plane and the broadening is proportional to the average number of steps in the random walk, i.e., the average number of semi-hard scatterings suffered along the wounded parton trajectory. This picture will be studied in more detail in Sec. 5.1.

An explicit formula for the transverse spectrum can be obtained by studying its Fourier transform, since all the convolutions in Eq. (4.2) turn into products and the sum over ν may be explicitly performed. To this purpose, we introduce the Fourier transform of the parton-parton scattering cross-section

$$\tilde{\sigma}(v; xx') = \int d^2k e^{i\mathbf{k} \cdot \mathbf{v}} \frac{d\sigma}{d^2k}(xx') .$$

Note that $\tilde{\sigma}(0; xx') = \sigma(xx')$ and that due to the azimuthal symmetry of $d\sigma/d^2k$, its Fourier transform depends only on the modulus, v , of \mathbf{v} . Then, the transverse spectrum (4.2) may be written as:

$$\frac{dW_h}{d^2p_t}(x, b, \beta) = \Gamma_h(x, b - \beta) \int \frac{d^2v}{(2\pi)^2} e^{-i\mathbf{p}_t \cdot \mathbf{v}} \widetilde{W}_h(v; x, b) , \quad (4.5)$$

where

$$\begin{aligned} \widetilde{W}_h(v; x, b) &= \sum_{\nu=1}^{\infty} \frac{1}{\nu!} \left[\int dx' \Gamma_A(x', b) \tilde{\sigma}(v; xx') \right]^{\nu} e^{-\int dx' \Gamma_A(x', b) \tilde{\sigma}(0; xx')} \\ &= e^{\int dx' \Gamma_A(x', b) \{\tilde{\sigma}(v; xx') - \tilde{\sigma}(0; xx')\}} - e^{-\int dx' \Gamma_A(x', b) \tilde{\sigma}(0; xx')} . \end{aligned} \quad (4.6)$$

An immediate consequence is that the transverse spectrum has a finite limit as $p_t \rightarrow 0$, even when a cutoff on the momentum exchange is used:

$$\left. \frac{dW_h}{d^2p_t} \right|_{\mathbf{p}_t=0}(x, b, \beta) = \Gamma_h(x, b - \beta) \int \frac{d^2v}{(2\pi)^2} \widetilde{W}_h(v; x, b) .$$

4.2 Expansion in the number of scatterings

We can obtain an expansion of \widetilde{W}_h in the number of the rescatterings suffered by the incoming parton by expanding Eq. (4.6) in powers of $\tilde{\sigma}$:

$$\begin{aligned}\widetilde{W}_h(v; x, b) &= \sum_{\nu=1}^{\infty} \widetilde{W}_h^{(\nu)}(v; x, b) \\ &= \sum_{\nu=1}^{\infty} \frac{1}{\nu!} \left[\left(\int dx' \Gamma_A(x', b) [\tilde{\sigma}(v; xx') - \tilde{\sigma}(0; xx')] \right)^{\nu} - \left(- \int dx' \Gamma_A(x', b) \tilde{\sigma}(0; xx') \right)^{\nu} \right].\end{aligned}\quad (4.7)$$

Coming back to the p_t space, the expansion of the transverse spectrum in number of scatterings reads:

$$\frac{dW_h}{d^2p_t}(x, b, \beta) = \sum_{\nu=1}^{\infty} \frac{dW_h^{(\nu)}}{d^2p_t}(x, b, \beta) = \sum_{\nu=1}^{\infty} \Gamma_h(x, b - \beta) \int \frac{d^2v}{(2\pi)^2} e^{-i\mathbf{p}_t \cdot \mathbf{v}} \widetilde{W}_h^{(\nu)}(v; x, b). \quad (4.8)$$

The series Eq. (4.7) can be obtained also by expanding $\widetilde{W}(v)$ around $v = 0$. Since the variable v is Fourier-conjugated to p_t , the expansion of the transverse spectrum, Eq. (4.8), will be valid at high p_t and we expect a breakdown of any truncation at sufficiently low momentum. Note that we can obtain this high- p_t expansion of the spectrum directly in p_t space by expanding the exponential in (4.2) and collecting the terms of the same order in σ . As an example, the first three terms, Eqs. (A.1), (A.2) and (A.6), can be found in the appendix. The first two, suitably symmetrized, were used in [2, 3, 4, 5] to explain the Cronin effect up to $\sqrt{s} = 39$ GeV/A. The study of this series is the subject of Sec. 4.3; numerical results up to $n = 3$ scatterings will be discussed in Sec. 5.1 and compared to the whole spectrum. In the appendix we will discuss the symmetrization of the terms of the series.

4.3 Cancellation of the divergences

All terms of the expansion (4.8) are divergent in the infrared region so that we need to cure them with the regulator p_0 . Nevertheless, the infrared divergences are already regularized to a large extent by the subtraction terms originated by the expansion of $\exp[-\langle n_A(x, b) \rangle]$ appearing in Eq. (4.2), namely by the constraint of probability conservation. This cancellation mechanism was observed also in Ref.[3] for the two-scattering term and in Ref.[13] in a different context.

It is instructive to look in detail how the subtraction works for the lower order terms of the expansion. We start by considering the case of a single rescattering ($\nu = 2$). To simplify the notation we write the elementary differential cross-section $d\sigma/d^2k$ as $\sigma(\mathbf{k})$, and notice that it depends only on the modulus, k , of the momentum. By expressing the

semi-hard cross-section as $\sigma = \int d^2k \sigma(\mathbf{k})$ the term of order σ^2 may be written as

$$\begin{aligned} \frac{dW_h^{(2)}}{d^2p_t}(x, b, \beta) &= \Gamma_h(x, b - \beta) \int \Gamma_A(x'_1, b) \Gamma_A(x'_2, b) dx'_1 dx'_2 d^2k_1 d^2k_2 \\ &\times \frac{\sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)}{2} \left[\delta^{(2)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_t) - \delta^{(2)}(\mathbf{k}_1 - \mathbf{p}_t) - \delta^{(2)}(\mathbf{k}_2 - \mathbf{p}_t) \right], \end{aligned} \quad (4.9)$$

where the first term in the square brackets represent two successive scatterings with no absorption. The two negative terms are the corrections induced by the expansion of the absorption factor $\exp[-\langle n_A(x, b) \rangle]$ of the single-scattering term, $\nu = 1$ in (4.2), and correspond to a single-scattering along with the effects of absorption in the initial or final state. The expression we obtained is symmetric in the integration variables \mathbf{k}_1 and \mathbf{k}_2 . The cutoff dependence is originated by the singular behavior of the integrand for $\mathbf{k}_1 \approx 0$ or for $\mathbf{k}_2 \approx 0$, since the δ -functions in the square brackets prevent the possibility of \mathbf{k}_1 and \mathbf{k}_2 being both zero at the same time. Because of the symmetry under the exchange $\mathbf{k}_1 \leftrightarrow \mathbf{k}_2$, to study the cutoff dependence of Eq. (4.9) it is enough to discuss the integration around $\mathbf{k}_1 = 0$. In the region $\mathbf{k}_1 \approx 0$ the term $\delta^{(2)}(\mathbf{k}_1 - \mathbf{p}_t)$ does not contribute, as long as \mathbf{p}_t is finite. The integration in \mathbf{k}_2 is done with the help of the δ -functions and one obtains

$$\int d^2k_1 \sigma(\mathbf{k}_1) \left[\sigma(\mathbf{p}_t - \mathbf{k}_1) - \sigma(\mathbf{p}_t) \right].$$

On the other hand, for $\mathbf{k}_1 \approx 0$, one may use the expansion

$$\sigma(\mathbf{p}_t - \mathbf{k}_1) \simeq \sigma(\mathbf{p}_t) - \sigma'(\mathbf{p}_t) \frac{\mathbf{p}_t \cdot \mathbf{k}_1}{p_t},$$

where $\mathbf{p}_t \cdot \mathbf{k}_1$ represents the scalar product of the two vectors, and $\sigma'(\mathbf{p}_t) = \frac{d}{d|\mathbf{p}_t|} \sigma(\mathbf{p}_t)$ depends only on the modulus of \mathbf{p}_t . One is left with

$$-\frac{\sigma'(\mathbf{p}_t)}{p_t} \int \mathbf{p}_t \cdot \mathbf{k}_1 \sigma(\mathbf{k}_1) d^2k_1 = 0,$$

where the vanishing result is due to the azimuthal symmetry of $\sigma(\mathbf{k}_1)$. The dominant contribution to the integral comes therefore from the next term in the expansion of $\sigma(\mathbf{k}_1 - \mathbf{p}_t)$, which goes as k_1^2 . Hence the resulting singularity is only logarithmic in p_0 , since $\sigma(\mathbf{k}) \sim k^{-4}$ as $k \rightarrow 0$. The subtraction terms, originated by the absorption factor $\exp[-\langle n_A(x, b) \rangle]$ in Eq. (4.9), have cancelled the singularity of the rescattering term almost completely. This feature is common to all the terms of the expansion (4.9).

Hereafter we consider in detail the term with two rescatterings:

$$\begin{aligned} \frac{dW_h^{(3)}}{d^2p_t}(x, b, \beta) &= \Gamma_h(x, b - \beta) \int \Gamma_A(x'_1, b) \Gamma_A(x'_2, b) \Gamma_A(x'_3, b) dx'_1 dx'_2 dx'_3 d^2k_1 d^2k_2 d^2k_3 \\ &\times \frac{\sigma(\mathbf{k}_1)\sigma(\mathbf{k}_2)\sigma(\mathbf{k}_3)}{6} \left[\delta^{(2)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{p}_t) \right. \\ &- \delta^{(2)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_t) - \delta^{(2)}(\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{p}_t) - \delta^{(2)}(\mathbf{k}_3 + \mathbf{k}_1 - \mathbf{p}_t) \\ &\left. + \delta^{(2)}(\mathbf{k}_1 - \mathbf{p}_t) + \delta^{(2)}(\mathbf{k}_2 - \mathbf{p}_t) + \delta^{(2)}(\mathbf{k}_3 - \mathbf{p}_t) \right]. \end{aligned} \quad (4.10)$$

The different δ -functions in Eq. (4.10) correspond to all the terms of order σ^3 in Eq. (4.2) and represent the triple scattering term together with all subtraction terms, induced by the expansion of the absorption factor $\exp[-\langle n_A(x, b) \rangle]$ of the double- and of the single-scattering terms. The expression has been symmetrized with respect to \mathbf{k}_1 , \mathbf{k}_2 and \mathbf{k}_3 and is singular for $\mathbf{k}_1 = 0$, $\mathbf{k}_2 = 0$ and $\mathbf{k}_3 = 0$. The δ -functions in (4.10) prevent the tree momenta to be close to zero at the same time, then we start by discussing the most singular configuration corresponding to two integration variables both close to zero. Given the symmetry of the integrand it is enough to study the integration region with $\mathbf{k}_1 \approx 0$, $\mathbf{k}_2 \approx 0$. In this region the terms $\delta^{(2)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_t)$, $\delta^{(2)}(\mathbf{k}_1 - \mathbf{p}_t)$ and $\delta^{(2)}(\mathbf{k}_2 - \mathbf{p}_t)$ do not contribute. The integrals on the transverse momenta are therefore written as

$$\int d^2 k_1 d^2 k_2 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) \left[\sigma(\mathbf{p}_t - \mathbf{k}_1 - \mathbf{k}_2) - \sigma(\mathbf{p}_t - \mathbf{k}_1) - \sigma(\mathbf{p}_t - \mathbf{k}_2) + \sigma(\mathbf{p}_t) \right]. \quad (4.11)$$

In the region $\mathbf{k}_1 \approx 0$, $\mathbf{k}_2 \approx 0$ one may use the expansion

$$\sigma(\mathbf{p}_t - \mathbf{k}) \simeq \sigma(\mathbf{p}_t) - \sigma'(\mathbf{p}_t) \frac{\mathbf{p}_t \cdot \mathbf{k}}{p_t} + \frac{1}{2} \left[\sigma''(\mathbf{p}_t) \frac{(\mathbf{p}_t \cdot \mathbf{k})^2}{p_t^2} - \sigma'(\mathbf{p}_t) \frac{(\mathbf{p}_t \times \mathbf{k})^2}{p_t^3} \right], \quad (4.12)$$

where $\mathbf{p}_t \times \mathbf{k}$ represents the vector product of \mathbf{p}_t and \mathbf{k} and $\sigma''(\mathbf{p}_t) = \frac{d^2}{d|\mathbf{p}_t|^2} \sigma(\mathbf{p}_t)$ depends only on the modulus of \mathbf{p}_t . All terms proportional to $\sigma(\mathbf{p}_t)$ cancel and all the terms linear in \mathbf{k} integrate to zero thanks to the azimuthal symmetry of $\sigma(\mathbf{k})$. Then one is left with

$$\begin{aligned} \int d^2 k_1 d^2 k_2 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) & \left\{ \frac{\sigma''(\mathbf{p}_t)}{2p_t^2} \left[(\mathbf{p}_t \cdot (\mathbf{k}_1 + \mathbf{k}_2))^2 - (\mathbf{p}_t \cdot \mathbf{k}_1)^2 - (\mathbf{p}_t \cdot \mathbf{k}_2)^2 \right] \right. \\ & \left. - \frac{\sigma'(\mathbf{p}_t)}{2p_t^3} \left[(\mathbf{p}_t \times (\mathbf{k}_1 + \mathbf{k}_2))^2 - (\mathbf{p}_t \times \mathbf{k}_1)^2 - (\mathbf{p}_t \times \mathbf{k}_2)^2 \right] \right\}, \end{aligned}$$

which simplifies to

$$\int d^2 k_1 d^2 k_2 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) \left\{ \frac{\sigma''(\mathbf{p}_t)}{p_t^2} (\mathbf{p}_t \cdot \mathbf{k}_1) (\mathbf{p}_t \cdot \mathbf{k}_2) - \frac{\sigma'(\mathbf{p}_t)}{p_t^3} (\mathbf{p}_t \times \mathbf{k}_1) (\mathbf{p}_t \times \mathbf{k}_2) \right\} = 0.$$

The result is again zero because of the azimuthal symmetry of $\sigma(\mathbf{k})$. Hence, all terms of the expansion (4.12) up to the second order in k do not contribute. All other terms linear in \mathbf{k}_1 or in \mathbf{k}_2 , which are obtained from the first terms in the square brackets in Eq. (4.11), do not contribute for the same reason, so the first term different from zero is at least of order $k_1^2 k_2^2$, and originates a square-logarithm singularity as a function of the regulator p_0 .

One may repeat the argument for the regions where only one of the integration variables is close to zero. We consider in detail the case $\mathbf{k}_1 \approx 0$ and $\mathbf{k}_2, \mathbf{k}_3$ both finite. In this region the term $\delta^{(2)}(\mathbf{p}_t - \mathbf{k})$ does not contribute to Eq. (4.10). The transverse momentum integrals are therefore

$$\begin{aligned} \int d^2 k_1 d^2 k_2 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) & \left\{ \sigma(\mathbf{p}_t - \mathbf{k}_1 - \mathbf{k}_2) - \sigma(\mathbf{p}_t - \mathbf{k}_1) - \sigma(\mathbf{p}_t - \mathbf{k}_2) + \sigma(\mathbf{p}_t) \right\} \\ & + \int d^2 k_1 d^2 k_3 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_3) \left\{ -\sigma(\mathbf{p}_t - \mathbf{k}_1) + \sigma(\mathbf{p}_t) \right\}. \end{aligned}$$

To study the singularity it is sufficient to keep the first two terms in the expansion of $\sigma(\mathbf{k})$ in 4.12. One obtains

$$\begin{aligned} & \int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) \\ & \times \left\{ \sigma(\mathbf{p}_t - \mathbf{k}_2) - \sigma'(\mathbf{p}_t - \mathbf{k}_2) \frac{(\mathbf{p}_t - \mathbf{k}_2) \cdot \mathbf{k}_1}{\mathbf{p}_t - \mathbf{k}_2} - \sigma(\mathbf{p}_t) + \sigma'(\mathbf{p}_t) \frac{\mathbf{p}_t \cdot \mathbf{k}_1}{p_t} - \sigma(\mathbf{p}_t - \mathbf{k}_2) + \sigma(\mathbf{p}_t) \right\} \\ & + \int d^2k_1 d^2k_3 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_3) \left\{ -\sigma(\mathbf{p}_t) + \sigma'(\mathbf{p}_t) \frac{\mathbf{p}_t \cdot \mathbf{k}_1}{p_t} - \sigma(\mathbf{p}_t) \right\}, \end{aligned}$$

which simplifies to

$$\int d^2k_1 d^2k_2 \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) \left\{ -\sigma'(\mathbf{p}_t - \mathbf{k}_2) \frac{(\mathbf{p}_t - \mathbf{k}_2) \cdot \mathbf{k}_1}{\mathbf{p}_t - \mathbf{k}_2} + 2\sigma'(\mathbf{p}_t) \frac{\mathbf{p}_t \cdot \mathbf{k}_1}{p_t} \right\} = 0.$$

As in the previous case one obtains a vanishing result thanks to the azimuthal symmetry of $\sigma(\mathbf{k})$. All integrations in the singular points induce therefore at most a square-logarithm singularity, as a function of the cutoff, in the term with $\nu = 3$ in Eq. (4.8).

The argument holds for the whole spectrum, as one may see by looking at its Fourier transform, Eq. (4.6). Indeed, to study the dependence of the inclusive spectrum on the regulator p_0 at a given p_t different from zero one needs to consider the first term in the square brackets only. The cutoff enters in the difference

$$\tilde{\sigma}(v) - \tilde{\sigma}(0) = \int \frac{d\sigma}{d^2k} [e^{i\mathbf{k} \cdot \mathbf{v}} - 1] d^2k = -v^2 \frac{\pi}{2} \int_{p_0}^{\infty} k^3 \frac{d\sigma}{d^2k} dk + \text{finite terms}$$

so that, also in this case, the divergence for $p_0 \rightarrow 0$ is only logarithmic.

5 Numerical results and discussion

In this section we discuss in detail, both qualitatively and quantitatively, the modifications induced by the rescatterings on the minijet inclusive transverse spectrum. We consider a proton-lead collision with center of mass energy $\sqrt{s} = 6$ TeV/A and impact parameter $\beta = 0$. In the numerical computations we used the leading order perturbative parton-parton cross-section with a mass regulator $m \equiv p_0$:

$$\frac{d\sigma}{d^2p}(xx') = k \frac{9\pi\alpha_s(Q)^2}{(p^2 + m^2)^2} \theta(xx's - 4(p^2 + m^2))$$

where k is the k -factor that simulates next-to-leading order corrections (we chose $k = 2$). The single-parton nuclear distribution function has been taken to be factorized in x and b :

$$\Gamma_A(x, b) = \tau_A(b) G(x, Q)$$

where τ_A is the nuclear thickness function and G is the proton distribution function. We evaluated the strong coupling constant and the nuclear distribution functions at a fixed scale $Q = m$. In the computations we used a hard-sphere geometry

$$\tau_A(b) = A \frac{3}{2\pi R^3} \sqrt{R^2 - b^2} \theta(R^2 - b^2)$$

where $R = 1.12A^{1/3}$ is the nuclear radius. For G we used the GRV98LO parameterization [28]. At low p_t the spectrum is obtained by computing numerically the Fourier transform in Eq. (4.5), but at high p_t the result begins to oscillate too much, and in that region the spectrum was computed by using the expansion in the number of scattering up to the three-scattering term (the formulae actually used, Eqs. (A.1), (A.4) and (A.8), are discussed in the appendix). We checked that the spectrum obtained by Fourier transformation matched smoothly the expansion.

5.1 Effects of rescatterings

In this section we discuss the projectile and the target transverse spectrum averaged over a given rapidity interval:

$$\frac{dW_h}{d^2p_t}(\beta, \eta_{min}, \eta_{max}) = \frac{1}{\eta_{max} - \eta_{min}} \int_{\eta \in [\eta_{min}, \eta_{max}]} dx d^2b \frac{dW_h}{d^2p_t}(x, b, \beta) , \quad (5.1)$$

where we approximated the pseudo-rapidity by $\eta = \log(x\sqrt{s}/p_0)$. The target spectrum, dW_A/d^2p_t , is obtained by interchanging h and A in Eq. (5.1). Note that now we are taking into account all possible rescatterings of the target, as well.

In fig.1 we compare the full transverse spectrum (solid line) with its expansion in the number of scatterings up to three scatterings (dotted and dashed lines). We show both the projectile and target minijet spectrum in a pseudo-rapidity region $\eta \in [3, 4]$ for the projectile and $\eta \in [-4, -3]$ for the target. Note that the rapidity is defined with reference to the projectile hadron direction of motion. The choice of a forward region (backward for the target) is done to enhance the effect of the rescatterings and to better discuss the deformation induced in the spectrum. Indeed, in those regions the average fractional momentum of an incoming parton is large, so that the number of available target partons is large and the probability of rescattering becomes large.

First, we look at the projectile spectrum. At high p_t the spectrum is enhanced with respect to the single scattering approximation because of the p_t broadening induced by the rescatterings. As p_t is further increased it approaches the single-scattering spectrum, as expected on general grounds when the p_t distribution of the elementary scattering follows a power law. This can be understood qualitatively by looking at the path in p_t -space followed by the incoming parton. Given a final large p_t , due to the leading divergences in Eq. (4.9), the leading processes to get that p_t with two semi-hard scatterings are a first scattering with momentum transfer $q_1 \approx p_0$ followed by a second one with $q_2 \approx p_t$ and vice-versa. For an analogous reason, the leading configuration to reach the final p_t with three scatterings is $q_1 \approx p_t$ plus $q_2 \approx q_3 \approx p_0$ and permutations. This sequence of three scatterings is less probable than the process with two scatterings as p_t increases because

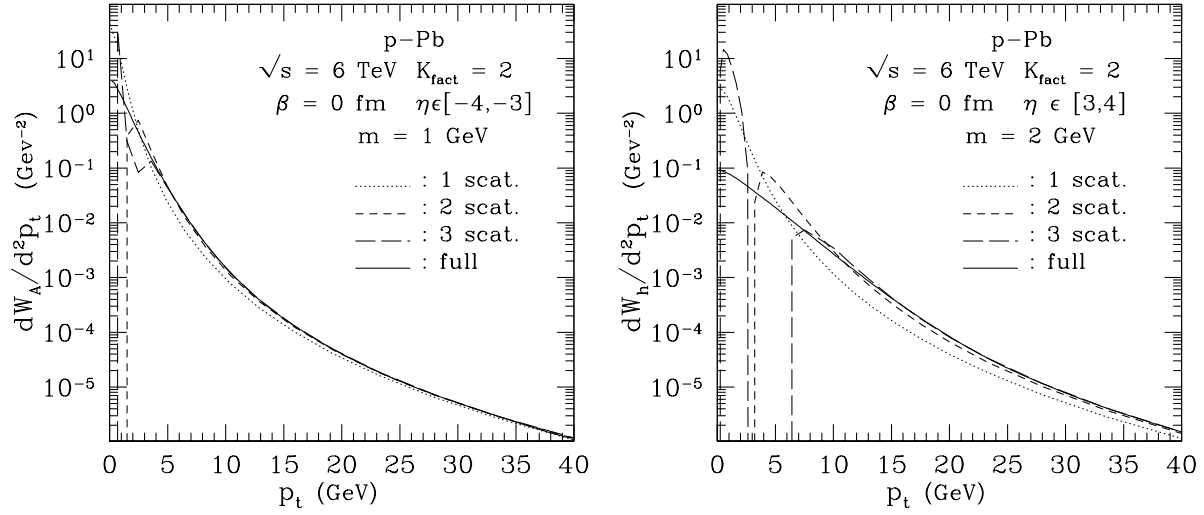


Figure 1: *Left:* Target p_t -spectrum for $\eta \in [-4, -3]$. *Right:* Projectile p_t -spectrum for $\eta \in [3, 4]$. The full transverse spectrum (solid line) is compared with the one-, two- and three-scattering approximations (viz., dotted, short-dashed and long-dashed lines).

the fraction of phase-space volume that this process occupies decreases much faster with p_t than in the two-scattering case. For an analogous reason also the relative importance of the two-scattering term with respect to the single-scattering term decreases as p_t increases. In conclusion as p_t increases the average number of scatterings per parton decreases, and eventually the spectrum is well described by the single-scattering approximation.

At intermediate p_t the average number of scatterings per parton increases and the shape of the spectrum is more and more distorted with respect to the single-scattering case. In fact, the fraction of phase-space available to the leading configuration of a multiple scattering process ($q_1 \approx p_t$, $q_2 \approx \dots \approx q_n \approx p_0$ and permutations) increases as p_t decreases. However, this is not the only mechanism at work. Indeed, in our computation each wounded parton is counted as one minijet in the final state, independently of the number of rescatterings. On the other hand, in the single-scattering approximation one identifies the number of minijets in the final state with the number of parton-parton collision. This leads to an overestimate of the jet multiplicity and to a divergence of the spectrum at $p_t = 0$ as p_0 goes to zero. Therefore at low p_t the minijet yield is more and more suppressed with respect to the single scattering approximation.

At very low transverse momentum $p_t \lesssim p_0$ a parton undergoes a large number of rescatterings, all with $q_i \approx p_0$. Hence, the parton is doing a random-walk in the transverse plane and the spectrum becomes flat as $p_t \rightarrow 0$ because the phase space becomes isotropically populated. This shows that at very low p_t multiple semi-hard scatterings are consistent with the random-walk model of Ref. [15], while at moderate and high- p_t the physical picture is rather different.

By comparing the results for the projectile and target transverse spectrum one sees that a projectile parton is traversing a very dense target and the effects of the rescatterings

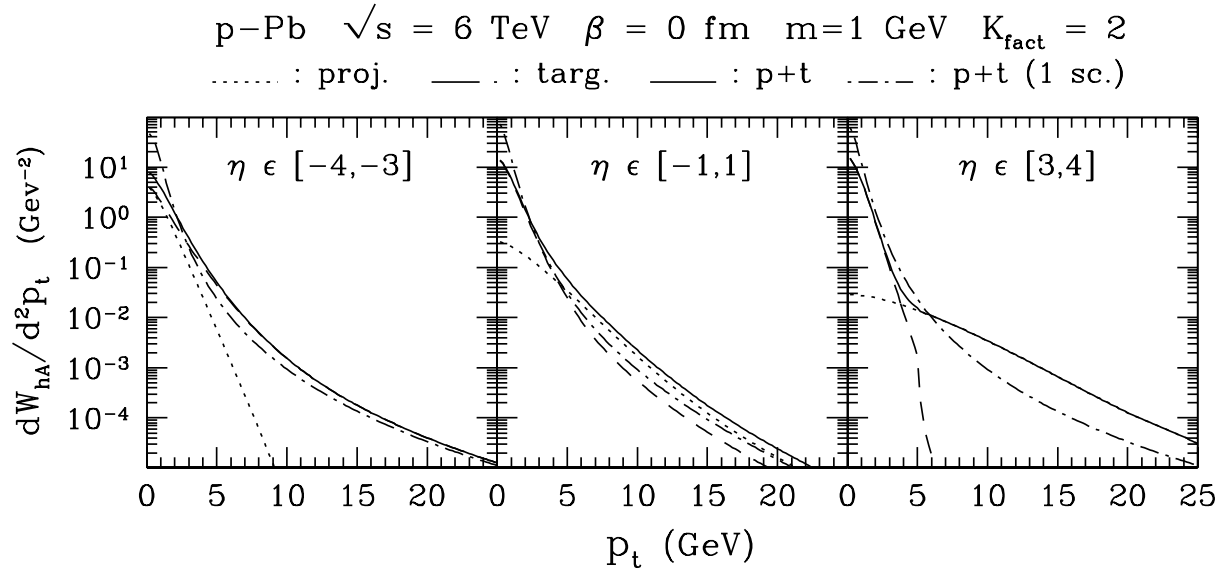


Figure 2: Projectile plus target p_t -spectrum (solid line) at different rapidities compared to the result of the one-scattering approximation (dot-dashed line). Also shown are the contributions of the projectile minijets (dotted line) and of the target minijets (dashed line).

are large. On the contrary, a target parton sees a rather dilute system, and its minijet spectrum does not differ too much from the single-scattering result, except at very low p_t . Moreover the changes induced by the rescatterings on integrated quantities, like those entering in the expression of the hadron-nucleus cross-section, are minimal. This is consistent with our approximation of not including rescatterings for the target partons to obtain analytic formulae for the hadron-nucleus cross-section. One can also see that the three-scattering approximation describes well the projectile spectrum for $p_t \gtrsim 15 \text{ GeV}$, while it breaks down completely at $p_t \lesssim 7 \text{ GeV}$, where it becomes negative. For the target spectrum the three-scattering approximation is not accurate for $p_t \lesssim 4 \text{ GeV}$.

5.2 Minijet inclusive transverse spectrum

In this section we study the minijet transverse spectrum resulting from the sum of the transverse spectra of the projectile and target wounded partons:

$$\frac{dW_{hA}}{d^2p_t}(\beta, \eta_{\min}, \eta_{\max}) = \frac{1}{\eta_{\max} - \eta_{\min}} \int_{\eta \in [\eta_{\min}, \eta_{\max}]} dx d^2b \left(\frac{dW_h}{d^2p_t}(x, b, \beta) + \frac{dW_A}{d^2p_t}(x, b, \beta) \right). \quad (5.2)$$

We analyze the spectrum in three rapidity regions, namely $\eta \in [-4, -3]$, $\eta \in [-1, 1]$ and $\eta \in [3, 4]$ (respectively “backward”, “central” and “forward” with reference to the projectile direction of motion). While the target partons basically do not suffer any rescattering in all three regions, the projectile partons undergoes many rescatterings in the forward region, some in the central region and basically no one backwards.

In Fig. 2 we show the spectrum (5.2) (solid line) and the contributions of the projectile and of the target (dotted and dashed lines, respectively). For comparison also the total

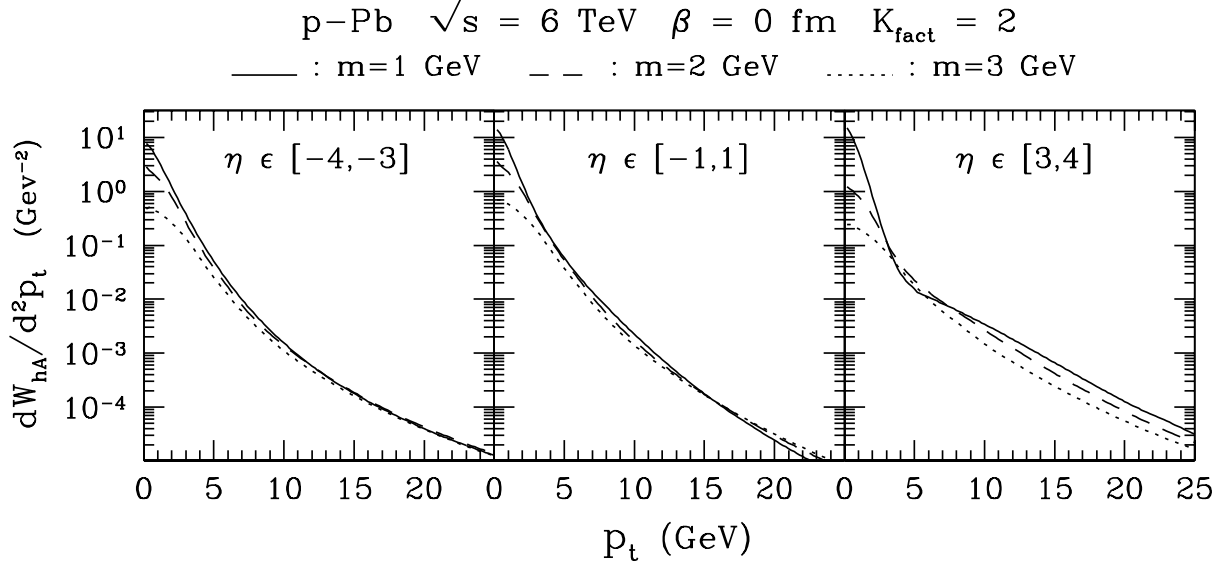


Figure 3: Regulator dependence of the projectile plus target p_t -spectrum at different rapidities for $m = 1, 2, 3$ GeV (viz., solid, dashed and dotted line).

spectrum obtained in the one-scattering approximation is plotted (dot-dashed line). The spectra are computed with a regulating mass $m = 1$ GeV.

In the backward region both the projectile and the target suffer mainly one scattering over all the p_t -range except at $p_t \sim 0$, and the spectrum is dominated almost everywhere by target minijets.

In central and forward regions the target jets still suffer basically one scattering over all the p_t range. On the contrary, the projectile crosses a denser and denser target and undergoes an average number of rescatterings that increases with pseudo-rapidity. This means that at low p_t the projectile spectrum is very reduced with respect to the one-scattering approximation, and the minijet yield may become negligible with respect to the minijet yield from target. The overall effect is that at low p_t the spectrum is dominated by minijet production from the target while at intermediate and high p_t it is dominated by minijet production from the projectile.

At very forward rapidities this effect becomes quite dramatic and the spectrum acquires a structured shape: it follows the inverse power behaviour of the single-scattering term at high p_t , it is concave at intermediate p_t because of the suppression of the projectile minijets and becomes convex again at low p_t , where the target begins to dominate.

In Fig.3 we study the dependence of the spectrum on the choice of the cutoff, and plotted the result for $m = 1, 2, 3$ GeV. The deformation of the spectrum decreases as the regulator increases (indeed, the average number of rescattering decreases) and for $m \gtrsim 3$ GeV it begins to become negligible.

The effects of the rescatterings are better displayed by studying the ratio of the full transverse spectrum and the single-scattering approximation:

$$R_\beta(p_t) = \frac{dW_{hA}/d^2p_t}{dW_{hA}^{(1)}/d^2p_t} = \frac{dW_{hA}/d^2p_t}{A_\beta dW_{pp}^{(1)}/d^2p_t}, \quad (5.3)$$

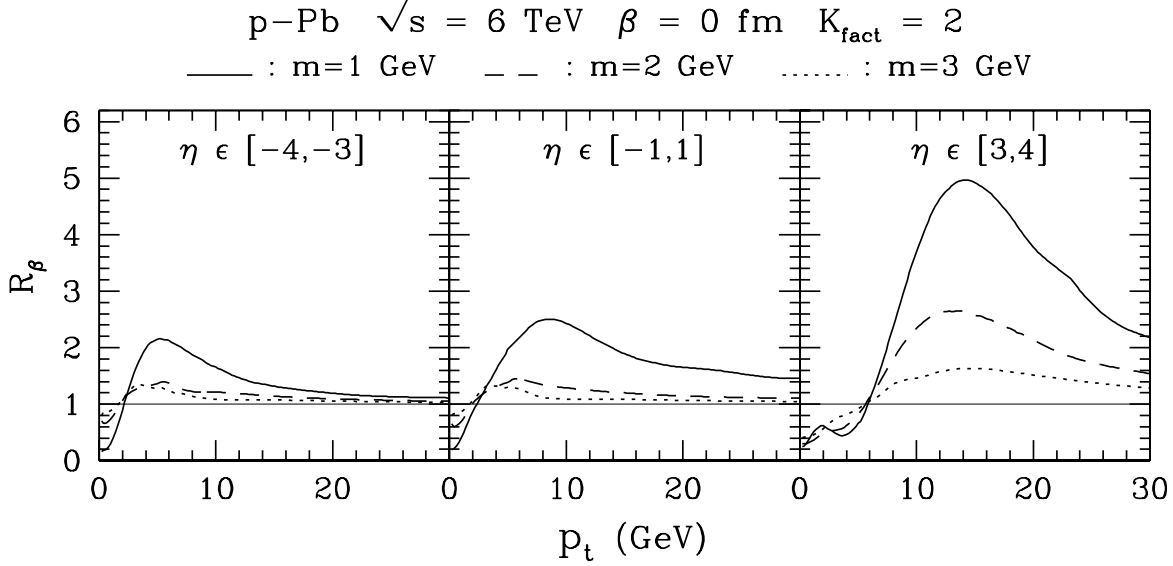


Figure 4: Ratio of the full projectile plus parton p_t -spectrum to the one-scattering approximation at different rapidities and for $m = 1, 2, 3 \text{ GeV}$ (viz., solid, dashed and dotted line).

where $A_\beta \simeq \int d^2b \tau_h(b - \beta) \tau_A(b)$ is the number of target nucleons interacting with the projectile at a given impact parameter.

In Fig.4 we plotted the ratio $R_\beta(p_t)$, which measures the Cronin effect for minijet production, computed with three different regulators $m = 1, 2, 3 \text{ GeV}$. At $m = 3 \text{ GeV}$ the effect of the rescatterings is rather small in all the three rapidity intervals, except at very low p_t , and doesn't affect the integrated quantities like the average number of minijets. As the regulating mass is decreased the rescatterings begin to show up, and lead to a big effect in the forward region.

The ratio $R_\beta(p_t)$ is characterized by three quantities: the momentum p_\times where the R_β crosses 1, the momentum p_M where it reaches the maximum and the height R_M of the maximum.

The sensitivity of p_\times on the cutoff decreases as the pseudo-rapidity increases. Loosely speaking, when the average number of scatterings is high, as it is the case at $p_t \simeq p_\times$, the jets loose memory of p_0 , which gives the order of magnitude of the typical momentum exchanged in each collision. p_M shows a slightly larger sensitivity on the regulator, since it lies in a region where the average number of scatterings is smaller. This behaviour is very different from the conclusions drawn by considering only the expansion up to two scatterings, where both p_\times and p_M are proportional to p_0 [5]. In fact, at low center of mass energies the two-scattering is a good approximation in all rapidity ranges, except may be very forward. However, it breaks down in any case at transverse momenta comparable to the regulator p_0 . Therefore, while most of the spectrum is well described by the two-scattering approximation, the behaviour of p_\times and p_M is not.

On the other hand, the height of the peak is much more sensitive to the cutoff, since its

leading term is roughly proportional to some power of the logarithm of the regulator:

$$\left[\frac{dW_{hA}}{d^2p_t} - \frac{dW_{hA}^{(1)}}{d^2p_t} \right]_{p_t=p_M} \underset{p_0 \rightarrow 0}{\sim} \left[\log \left(\frac{p_M^2}{p_0^2} \right) \right]^{\langle n_{resc}(p_M) \rangle}$$

Since p_M is not very large, the average number of rescatterings at that value of the transverse momentum, $\langle n_{resc}(p_M) \rangle$, is much greater than one and the sensitivity of R_M on p_0 is high. At high p_t the average number of rescatterings tends to zero, so the sensitivity of the R_β on p_0 decreases and disappears at very large transverse momenta.

Note that the peak is located in a p_t -region, where soft interactions (which have been disregarded in our approach) are expected to be negligible, therefore in that region our perturbative computations should describe almost completely the spectrum. Following Ref. [5] we might interpret p_0 as the momentum scale at which the interaction deviates from the perturbative computations. With this interpretation p_0 would acquire a physical meaning: though physics doesn't know about the artificial subdivision in hard and soft interactions, it is a well defined question to ask up to what scale are the perturbative computations good. If the collision dynamics would be determined by parton multiple elastic scatterings alone, then the measure of the height of the peak would be a way of measuring p_0 .

On the other hand, the sensitivity of R_β on p_0 is rather signaling a lack in our description of the dynamics underlying the hadron-nucleus collision. We expect that such a sensitivity will be considerably reduced when including in the dynamics also the gluon radiation emitted by the multiply scattering partons. Some of the effects of the radiation on the transverse spectrum might be however described by the parameter p_0 in the model where the radiation is neglected. Since the inclusion of gluon radiation in the dynamics would introduce new physical scales, like the radiation formation time, related to the energy of the collision and the nuclear size, we would expect in any case that the value of p_0 will depend on \sqrt{s} and A .

6 Conclusions

The purpose of the present article is to draw the attention to some of the advantages of studying hadron-nucleus semi-hard interactions at the LHC. As in the case of lower energies, hA interactions represent an important intermediate step to relate hh and AA reactions, being much simpler to understand as compared with the latter. Moreover, even at higher energies, like those obtainable at RHIC and LHC, in hA collisions we don't expect the formation of a dense and hot system, like the quark-gluon plasma, so that one can study directly the nuclear modification of the dynamics without the need of disentangling the effects of the structure of the target and those due to the formation and evolution of the dense system. Hadron-nucleus interactions represent therefore the baseline for the detection and the study of the new phenomena peculiar to AA collisions.

We faced the problem of unitarity corrections to the semi-hard cross-section by including explicitly semi-hard parton rescatterings in the collision dynamics, exploiting the self-shadowing property of the semi-hard interactions. In the interaction mechanism we took into account just elastic parton-parton collisions, while we neglected the production

processes at the partonic level (e.g., all $2 \rightarrow 3$ etc. elementary partonic processes), whose inclusion represents a non-trivial step in our approach and deserves further study.

Contrary to the case of AA collisions, we have been able to obtain closed analytic expressions for the semi-hard hA cross-section, Eq. (3.7). To that purpose a crucial assumption has been to consider the hadron as a dilute system, so that rescatterings of nuclear partons can be neglected, while rescatterings of the projectile are fully taken into account. In our expressions we have disregarded correlations in the nuclear multi-parton distributions, whose effect may be nevertheless studied in a straightforward way within the present functional approach.

We have then focused on the inclusive minijet transverse spectrum at fixed impact parameter, Eq. (4.5), which is influenced in a more direct way by the rescatterings. The modifications of the transverse spectrum induced by the semi-hard rescatterings of the projectile partons is emphasized in the ratio $R_\beta(p_t)$, Eq.5.3, defined as our p_t spectrum divided by the impulse approximation. In particular, we have evaluated it at $\beta = 0$ for different values of the regulator p_0 . The results are described by the values of p_\times (defined by $R_\beta(p_\times) = 1$), p_M (which is the value of p_t that maximizes the ratio) and R_M (which is the maximum of R_β). We obtain the both p_\times and p_M depend weakly on p_0 , while R_M has, on the contrary, a strong dependence on p_0 also when the regulator is rather small. Therefore, the results for the spectrum allow also to identify the limits of the picture of the dynamics considered in this paper. Analogously to the average transverse energy and the number of minijets in AA collisions [21], some of the features of R_β , like p_\times and p_M , show a tendency towards a limiting value at small p_0 . All these quantities depend therefore only marginally on details of the dynamics which have not been taken into account in the present approach. On the contrary, the limits of the simplified picture of the interaction show up in R_M . Because of the strong dependence of R_M on p_0 , in order to describe the spectrum one needs in fact to fix experimentally the value of p_0 by measuring R_M . This feature might be not so unpleasant, because if one limits the analysis to the inclusive transverse spectrum of minijets in hA collisions, all the effects which are not taken into account in the interaction (like the gluon radiation in the elementary collision process) are summarized by the value of a single phenomenological parameter. However this feature will not hold any further if one had to evaluate more differential properties of the produced state, which can be properly discussed only after introducing explicitly further details in the description of the elementary interaction process.

The experimental measure of the Cronin effect in minijet production in hA collisions would be therefore of major importance: it would allow to establish the correctness of the whole approach here described and it would represent the basis for a deeper insight in the semi hard interaction dynamics both for hA and AA collisions.

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A Expansion in the number of collisions

For the numerical computation of the high- p_t expansion of the minijet spectrum in the number of scatterings suffered by a projectile parton it is convenient to implement the subtraction of the IR divergences directly in the integrand. In this way the Monte-Carlo integrations, which we use because of the high dimensionality of the phase space (in particular for three or more scatterings), work at their best. In fact, Eqs. (4.9) and (4.10) are not suited for numerical implementation due to the delta functions. The basic property that allowed the cancellation of the divergence in the integrand was the symmetry under exchanges of the integration variables. Unfortunately after using the delta-functions to perform on of the integrals, one obtains in general non-symmetric expressions.

The goal of this appendix is to study how to symmetrize each term of the expansion of the transverse spectrum. We will discuss them in detail up to the three-scattering term, but the techniques discussed can be applied also to the generic term in the expansion. For simplicity, we will use the following notation, already introduced in the main text:

$$\sigma(\mathbf{k}) = \frac{d\sigma}{d^2k}(xx') .$$

A.1 One-scattering term

The one-scattering term doesn't include any subtraction term, so that we don't need to symmetrize it. It is simply given by

$$\frac{dW_h^{(1)}}{d^2p_t}(x, b, \beta) = \Gamma_h(x, b - \beta) \int dx' \Gamma_A(x', b) \sigma(\mathbf{p}_t) , \quad (\text{A.1})$$

and corresponds to the result one obtains by considering just disconnected parton collisions and neglecting parton rescatterings. It corresponds also to modeling the hadron-nucleus collision as a superposition of hadron-nucleus collisions.

A.2 Two-scattering term

The two-scattering term is given by Eq. (4.9), and we need to perform one integration over \mathbf{k}_1 or over \mathbf{k}_2 to dispose of the δ -functions. By calling simply \mathbf{q} the remaining integration variable we obtain

$$\begin{aligned} \frac{dW_h^{(2)}}{d^2p_t}(x, b, \beta) &= \Gamma_h(x, b - \beta) \int \Gamma_A(x'_1, b) \Gamma_A(x'_2, b) dx'_1 dx'_2 \\ &\times \int d^2q \left[\sigma(\mathbf{q}) \sigma(\mathbf{p}_t - \mathbf{q}) - 2\sigma(\mathbf{q}) \sigma(\mathbf{p}_t) \right] \end{aligned} \quad (\text{A.2})$$

As discussed in Section 4.3, the negative term in the expression above subtracts the leading inverse power divergence in the integrand leaving only a logarithmic divergence. However, the cancellation happens only after performing the integral over \mathbf{q} , which may

be a difficult result to achieve numerically (actually this is not a problem for the two-scattering term, due to the low dimensionality of the integral, but becomes a big issue from three scatterings on).

There are two divergences to be subtracted: one in $\mathbf{q} \sim 0$ and the other in $\mathbf{q} \sim \mathbf{p}_t$, but the subtraction term is divergent just in $\mathbf{q} \sim 0$, and the cancellation of the inverse power singularities is obtained only after performing the integration over \mathbf{q} . To allow the numerical integration to do a better and faster job, we want that the divergences in the convolution term and in the subtraction term be cancelled directly in the integrand. This is obtained by symmetrizing the integrand with respect to an interchange of the two singularities in the convolution term. Let's introduce therefore an operator that performs the interchange of the two singularities:

$$\mathbb{T} : \mathbf{q} \rightarrow \mathbf{p}_t - \mathbf{q} ,$$

so that

$$\mathbb{T} \int d^2 q f(\mathbf{q}) = \int d^2 q f(\mathbf{p}_t - \mathbf{q}) .$$

Note that the change of variables operated by \mathbb{T} has unit Jacobian and that $\mathbb{T}^2 = \mathbb{I}$. Then, we define the symmetrized two-scattering term as

$$\left. \frac{dW_A^{(2)}}{d^2 p_t} \right|_{sym} = \mathbb{S}^{(2)} \frac{dW_h^{(2)}}{d^2 p_t} ,$$

where we introduced the symmetrization operator

$$\mathbb{S}^{(2)} = \frac{1}{2}(\mathbb{I} + \mathbb{T}) . \quad (\text{A.3})$$

The result is:

$$\begin{aligned} \left. \frac{dW_A^{(2)}}{d^2 p_t} \right|_{sym} (x, b, \beta) &= \Gamma_h(x, b - \beta) \int \Gamma_A(x'_1, b) \Gamma_A(x'_2, b) dx'_1 dx'_2 \\ &\times \int d^2 q \left[\sigma(\mathbf{q}) \sigma(\mathbf{p}_t - \mathbf{q}) - \sigma(\mathbf{q}) \sigma(\mathbf{p}_t) - \sigma(\mathbf{p}_t - \mathbf{q}) \sigma(\mathbf{p}_t) \right] . \quad (\text{A.4}) \end{aligned}$$

Note that the first term in (A.4) describes two subsequent scatterings with total transverse momentum p_t and is the naive pQCD result. The two negative terms are the absorption terms induced by probability conservation. The two IR divergences of the first term are canceled by these two subtraction terms: as $\mathbf{q} \rightarrow \mathbf{0}$ by the first one and as $\mathbf{q} \rightarrow \mathbf{p}_t$ by the second one. The remaining linear singularity gives a zero contribution because it is odd in a neighborhood of $\mathbf{q} = 0$ and $\mathbf{q} = \mathbf{p}_t$ so that only the logarithmic divergence remain. Note that now the two divergences are subtracted directly in the integrand, which was the goal of the symmetrization procedure.

Eq. (A.4) is the expression that we use in the numerical computations of the transverse spectrum at high p_t . It could have been guessed directly from Eq. (A.2), but the use of the symmetrization operator (A.3) will facilitate the discussion of the more complicated three scattering term.

A.3 Three-scattering term

To prepare the ground for the treatment of the three-scattering term, we note that \mathbb{T} generates the group of the permutations of the two singularities $\mathbf{q} \sim 0$ and $\mathbf{q} \sim \mathbf{p}_t$; this is called the symmetric group of order 2 and indicated as $S_2 = \langle \mathbb{T} \rangle = \{\mathbb{I}, \mathbb{T}\}$, where $\langle \mathbb{T} \rangle$ means “generated by \mathbb{T} ”. It’s then easy to see that we can construct the symmetrizing operator (A.3) by summing all the elements of S_2 and by dividing by its cardinality.

From Eq. (4.10), after exploiting the δ -functions, the three-scattering term reads

$$\frac{dW_h^{(3)}}{d^2p_t}(x, b, \beta) = \Gamma_h(x, b - \beta) \int \Gamma_A(x'_1, b) \Gamma_A(x'_2, b) \Gamma_A(x'_3, b) dx'_1 dx'_2 dx'_3 \quad (\text{A.5})$$

$$\times \frac{1}{3!} \int d^2q d^2r [\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) - 3\sigma(\mathbf{q})\sigma(\mathbf{p}_t - \mathbf{q})\sigma(\mathbf{p}_t) + 3\sigma(\mathbf{q})\sigma(\mathbf{r})\sigma(\mathbf{p}_t)] . \quad (\text{A.6})$$

Following the general analysis previously done at the end of the last paragraph, we observe that in (A.6) in absence of the cutoff we would have four divergences, i.e:

$$\mathbf{q} \sim 0, \quad \mathbf{r} \sim 0, \quad \mathbf{p}_t - \mathbf{q} - \mathbf{r} \sim 0, \quad \mathbf{p}_t - \mathbf{q} \sim 0 \quad (\text{A.7})$$

Then, to write the symmetrized three-scattering term, we need to consider the group S_4 of the permutations of these four divergences, which has $4! = 24$ elements:

$$\mathcal{P}_{Bsym}^{(3)} = \mathbb{S}^{(3)} \mathcal{P}_B^{(2)}$$

where

$$\mathbb{S}^{(3)} = \frac{1}{24} \sum_{\mathbb{T} \in S_4} \mathbb{T}$$

When applying this operator to the three-scattering term the resulting expression has 49 terms and is too long to be discussed here. To have an idea of the result, we will consider only the subgroup S_3 given by the permutations of the first three divergences in (A.7), which are the divergences that appear in the first term of (A.6), i.e. the naive three-scattering term. After the symmetrization it will be immediate to check that all the “single” divergences cancel explicitly in the integrand, while “double” divergences cancel only after performing the integrations over the transverse momenta. We call “single” divergence a point (\mathbf{q}, \mathbf{r}) such that only one of the expressions in (A.7) is near zero, and “double” divergence a point such that two of these terms are nearly zero. For example $\{\mathbf{q} \sim 0; \mathbf{r} \not\sim 0, \mathbf{p}_t, \mathbf{p}_t - \mathbf{q}\}$ and $\{\mathbf{q} \sim 0; \mathbf{r} \sim \mathbf{p}_t\}$ are respectively a single and a double divergence.

The first step is the definition of the operators that exchange the three singularities:

$$\mathbb{T}_1 : \begin{cases} \mathbf{q} \rightarrow \mathbf{r} \\ \mathbf{r} \rightarrow \mathbf{q} \end{cases} \quad \mathbb{T}_2 : \begin{cases} \mathbf{q} \rightarrow \mathbf{p}_t - \mathbf{q} - \mathbf{r} \\ \mathbf{r} \rightarrow \mathbf{r} \end{cases} \quad \mathbb{T}_3 : \begin{cases} \mathbf{q} \rightarrow \mathbf{q} \\ \mathbf{r} \rightarrow \mathbf{p}_t - \mathbf{q} - \mathbf{r} \end{cases}$$

Note that they are idempotent: $\mathbb{T}_i = \mathbb{I}$. Next, we observe that the group S_3 of the permutations of the three singularities is made of $3! = 6$ objects, and that

$$S_3 = \langle \mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3 \rangle = \{\mathbb{T}_0, \mathbb{T}_1, \mathbb{T}_2, \mathbb{T}_3, \mathbb{T}_4, \mathbb{T}_5\} ,$$

where $\mathbb{T}_0 = \mathbb{I}$, $\mathbb{T}_4 = \mathbb{T}_1\mathbb{T}_2$ and $\mathbb{T}_5 = \mathbb{T}_1\mathbb{T}_3$, so that the reduced symmetrizing operator is

$$\mathbb{S}_{red}^{(3)} = \frac{1}{3!} \sum_{i=0}^5 \mathbb{T}_i .$$

Finally one can write the partially symmetrized three-scattering probability:

$$\begin{aligned} \left. \frac{dW_A^{(3)}}{d^2p_t} \right|_{sym} (x, b, \beta) &= \mathbb{S}_{red}^{(3)} \frac{dW_A^{(3)}}{d^2p_t} (x, b, \beta) = \\ &= \Gamma_h(x, b - \beta) \int \Gamma_A(x'_1, b) \Gamma_A(x'_2, b) \Gamma_A(x'_3, b) dx'_1 dx'_2 dx'_3 d^2k_1 d^2k_2 d^2k_3 \\ &\quad \times \frac{1}{3!} \int dx' d^2q d^2r \left[\sigma(\mathbf{q}) \sigma(\mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) \right. \\ &\quad - \frac{1}{2} \sigma(\mathbf{q}) \sigma(\mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{q}) + \frac{1}{2} \sigma(\mathbf{q}) \sigma(\mathbf{p}_t - \mathbf{q}) \sigma(\mathbf{p}_t) \\ &\quad - \frac{1}{2} \sigma(\mathbf{q}) \sigma(\mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{r}) + \frac{1}{2} \sigma(\mathbf{p}_t - \mathbf{r}) \sigma(\mathbf{q}) \sigma(\mathbf{p}_t) \\ &\quad - \frac{1}{2} \sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) \sigma(\mathbf{q}) \sigma(\mathbf{p}_t - \mathbf{q}) + \frac{1}{2} \sigma(\mathbf{p}_t - \mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{q}) \sigma(\mathbf{p}_t) \\ &\quad - \frac{1}{2} \sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) \sigma(\mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{r}) + \frac{1}{2} \sigma(\mathbf{p}_t - \mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) \sigma(\mathbf{p}_t) \\ &\quad - \frac{1}{2} \sigma(\mathbf{r}) \sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) \sigma(\mathbf{q} + \mathbf{r}) + \frac{1}{2} \sigma(\mathbf{r}) \sigma(\mathbf{q} - \mathbf{r}) \sigma(\mathbf{p}_t) \\ &\quad \left. - \frac{1}{2} \sigma(\mathbf{q}) \sigma(\mathbf{p}_t - \mathbf{q} - \mathbf{r}) \sigma(\mathbf{q} + \mathbf{r}) + \frac{1}{2} \sigma(\mathbf{q}) \sigma(\mathbf{q} - \mathbf{r}) \sigma(\mathbf{p}_t) \right] . \quad (\text{A.8}) \end{aligned}$$

Analogously to what has been done for the two-scattering term, one can see by inspection that the four single divergences (A.7) explicitly cancel in the integrand, while double divergences cancel only after performing the integrations over q and r . By considering all four singularities, and by using the whole S_4 group we would get explicit cancellation of both “single” and “double” divergences directly in the integrand. Nonetheless, the partial symmetrization is enough to get satisfactory numerical results.

In conclusion, to compute numerically the expansion of the transverse minijet spectrum in the number of scatterings one has to fully exploit the symmetry properties of each term, in such a way that all the divergences get cancelled directly in the integrand. This is crucial to obtain a good numerical precision and to speed up the computation of the terms with three or more scatterings. In this appendix we developed a general technique to perform such a symmetrization.

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